Z Condensation
and
Standard Model Baryogenesis

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(February 1, 2008)

The standard model satisfies Sakharov's conditions for baryogenesis but the CP violation in the KM matrix appears too small to account for the observed asymmetry. In this letter we explore a mechanism through which CP violation can be greatly amplified. First CP is *spontaneously* broken through dynamical effects on bubble walls, and the two CP conjugate phases grow through phase ordering. Direct competition between macroscopic regions of both phases then amplifies the microscopic CP violation to a point where one of the two phases predominates. This letter is devoted to a demonstration that spontaneous CP violation may indeed occur on propagating bubble walls via the formation of a condensate of the longitudinal Z boson.

A fascinating development in particle theory over the last few years has been the realisation that the standard model, possibly with a minimal extension of the Higgs sector, has the potential to produce the observed matter-antimatter asymmetry in the universe. At the heart of the mechanism is the remarkable anomaly structure of the Weinberg-Salam theory, which ties changes in the topology of the gauge and Higgs fields to changes in the baryon number of the universe [1] If these baryon number violating processes were somehow biased at the electroweak phase transition, and subsequently turned off, an asymmetry would be generated rather naturally.

Interest in this idea grew when explicit mechanisms producing the required bias were proposed, in minimal extensions of the standard model involving extra Higgs fields (for reviews see [2,3]). Two simple arguments seem to exclude the minimal standard model. The CP violation due to the Kobayashi-Maskawa matrix is very small, suppressed by many powers of light quark masses and mixing angles. A naive estimate assuming analyticity in fermion masses indicates that CP violation should enter in the combination

$$d_{CP} = J\Pi_{i\neq j}(m_i^2 - m_j^2)\Lambda^{-12} \tag{1}$$

where the product is over like charged quarks, and $J \sim 10^{-5}$ is a product of KM angles. Taking the scale Λ to be the temperature $T \sim 100$ GeV, one finds $d_{CP} \sim 10^{-19}$ leading to a baryon symmetry well below the required value $\sim 10^{-10}$. Second, the requirement that the anomalous processes be suppressed after the phase transition imposes an upper bound on the mass of the Higgs boson - estimated from the one loop potential [4] to be $M_H < 35$ GeV, in conflict with the LEP bound $M_H > 60$ GeV. But recent lattice studies [5] indicate that the electroweak transition is far more strongly first order, relaxing the upper bound on M_H, perhaps up to 100 GeV.

The prospect of calculating the baryon asymmetry in terms of known experimental parameters make the search for a mechanism which amplifies (1) attractive. Recently, Farrar and Shaposhnikov [6] pointed out one very interesting mechanism involving very low momentum quasiparticles in the quark plasma. In their calculation, the scale Λ is provided by a light quark mass, enormously amplifying d_{CP}. However it appears that when damping due to strong scattering is included a negligible asymmetry results [7,8].

One might conclude that generating the observed asymmetry in the standard model is hopeless. We believe this is premature, and in this letter propose a new mechanism through which CP violation in the KM matrix may be greatly amplified in the standard model (or extensions such as the minimal supersymmetric version) through macroscopic physical effects qualitatively different than those previously considered.

There is another very *large* number involved in the electroweak transition, namely the horizon R_H in units of T^{-1}. The electroweak phase transition proceeds via bubble nucleation and growth. Calculations based on the perturbative potential show that bubbles fill space when their typical radius is $\sim 10^{-4} - 10^{-5} R_H$. If the phase transition is more strongly first order, this could increase to a value closer to $R_H \sim 10^{16} T^{-1}$.

Our scenario makes use of this as follows. As bubbles form and grow, collisions with fermions in the plasma lead to a CP violating instability on the bubble wall, namely a condensate of longitudinal Z bosons. The Z field can point either 'out' or 'in' (Figure 1). Patches of each phase initially cover each bubble, separated by one dimensional phase boundaries, which are in fact Z-magnetic flux tubes. As the bubble expands, small patches shrink away under the tension of the phase boundary, while larger patches grow with the bubble. The long time available before bubbles collide allows each bubble to be covered by *macroscopic* regions of the 'in' and the 'out' phases. Even if each bubble becomes completely ordered, macroscopic regions of both phases are placed in competition when large bubbles collide as the transition completes.

Now as a given phase moves through the plasma, its

terminal velocity is given by $v_t = \frac{\Delta P}{\Gamma}$ where ΔP is the pressure difference between the false and true vacua and where the frictional drag/unit area is given by $F_{\rm drag} = \Gamma v$. As a result of the CP violation in the KM matrix, we expect the 'in' and 'out' phases to feel slightly different Γ's, $\frac{\Gamma_+ - \Gamma_-}{\Gamma_+ + \Gamma_-} \sim d_{CP}$. It follows that the two phases move through the plasma with slightly different velocities $|v_+ - v_-| \sim d_{CP}$. Now consider the neighbourhood of a phase boundary. When the bubbles are large enough, we may take them to be planar as far as the local dynamics of the phase boundary go. As the bubbles grow, the 'out' phase begins to bulge over the 'in' phase, the height of the bulge being given by $\sim d_{CP} t$. After a time of order one expansion time, this becomes comparable to the scale which enters in determining the shape of the bubble wall, $l = \frac{\sigma}{\Delta P}$, where σ is the surface tension. The bulge then gives rise to a net tangential force on the phase boundary, causing it to sweep across the bubble at a velocity of the order of the bubble wall velocity. Thus the faster moving phase will (again in a time of the order of an expansion time), completely overtake the slower moving phase [15]. In this scenario, as the electroweak phase transition nears completion, bubble surfaces of one CP violating variety predominate. Notice that the only place the small CP-violating parameter comes in is in ensuring that the 'out' and 'in' phases expand at slightly different rates.

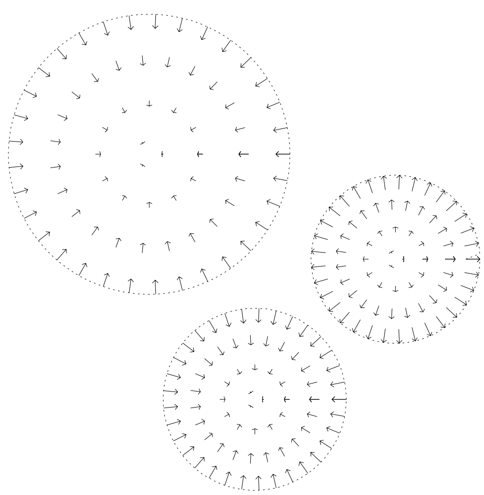

FIG. 1. Bubbles of the broken symmetry phase growing at the electroweak transition, with a condensate of longitudinal Z bosons on their surfaces. When large bubbles collide, 'in' and 'out' phases are placed in contact, leading to competition between macrocopic phases, which greatly amplifies CP violating effects.

The longitudinal Z condensate leads straightforwardly to the production of a baryon asymmetry through both 'local' [9,10] and 'nonlocal' mechanisms [11,12], because it couples to fermions in exactly the same way that the CP violating phase θ does in two-Higgs scenarios.

The remainder of this letter will be devoted to a demonstration that longitudinal Z condensate can form on propagating bubble walls. The longitudinal Z may be defined by the formula $i\varphi^* \overline{\mathcal{D}}_\mu \varphi \equiv -2 g_A \phi^2 Z_\mu$, where $\phi = \sqrt{2\varphi^\dagger \varphi}$ and $g_A = \frac{1}{4}(g_1^2 + g_2^2)^{\frac{1}{2}}$, with g_1, g_2 the $U(1)_Y$ and $SU(2)_W$ gauge couplings. We now consider the classical field equation for Z_μ, in background of a bubble wall propagating through the plasma. In the wall rest frame we take the Higgs field to have a fixed profile $\phi(z)$, its width L_w being of order the inverse of the Higgs mass m_H in the broken phase. The wall is treated as planar, which is a good approximation once a bubble gets large. We consider a Z field $Z^\mu = (0,0,0,Z(z))$. We may consistently drop the $W^\pm$ boson and photon fields, because the fermion currents induced by the Z are neutral and so only source the Z. Likewise the Z makes no contribution to the $SU(2)$ currents in the absence of a W field.

Next, we drop spatial derivative terms, which are suppressed by powers of m_H/m_Z, and m_H/m_{top} relative to the Z mass term and the top quark currents which dominate. The equation of motion then becomes

$$\ddot{Z} = J_Z(Z) + J_F(Z) \tag{2}$$

where $J_Z(Z) = -4 g_A^2 \phi^2 Z$ is the current carried by the Z condensate and $J_F(Z) = g_A \overline{\psi} \gamma^3 \gamma^5 \psi + g_V \overline{\psi} \gamma^3 \psi$ is that carried by the fermions.

In the absence of the $W^\pm$ bosons, CP invariance holds, and so $Z \to -Z$ is a symmetry [16]. Thus there is a solution $Z = J_F = 0$. But we shall show that this is unstable, so that a nonzero Z current develops. We expect this instability to lead to one of two stable solutions, related by $Z \to -Z$, in which $J_F(Z)$ and $J_Z(Z)$ are equal and opposite so that the net current is zero. The condition for an instability is simply that $m_{eff}^2 = 4 g_A^2 \phi^2 - (\partial J_F / \partial Z)|_{Z=0} < 0$.

The fermion current J_F is calculated by solving the time independent Dirac equation in the presence of ϕ and Z condensates. We have used the WKB approximation as a guide to the behaviour expected, and checked it against results obtained with an exact linear response function.

The relevant Dirac equation is

$$(i\gamma^\mu(\partial_\mu + i Z_\mu (g_V + g_A \gamma^5)) - m(z))\psi = 0 \tag{3}$$

where $m(z) = y\phi(z)$ with y the Yukawa coupling We ignore for the moment the one loop thermal contributions to the fermion self energies. From (3) the following dispersion relation is found

$$\omega_\pm = \sqrt{p_\perp^2 + (\sqrt{(p_z - g_V Z)^2 + m^2(z)} \mp g_A Z(z))^2} \tag{4}$$

for the two eigenstates, which have spin $S^z = \pm\frac{1}{2}$ in a frame where $p_\perp = 0$. In the rest frame of the wall, both the energy ω and the transverse momentum $p_\perp$ are conserved, and the local value of the momentum p_z varies according to (4). It is related to $p_{z,-\infty}$, the value of the momentum at $z = -\infty$, by the formula $E = \sqrt{p_\perp^2 + p_{z,-\infty}^2}$.

Regions for which $p_{z,-\infty} < m(z) \mp g_A Z$ are classically disallowed: the $S^z = \pm\frac{1}{2}$ excitations see a 'barrier' $m(z) \mp g_A Z(z)$ respectively (Figure 2).

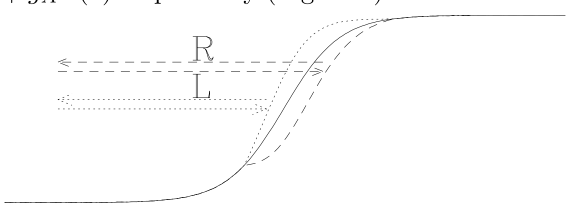

FIG. 2. Profile of a bubble wall, showing the barriers seen by particles with spins S^z in the positive and negative z directions. Positive spin particles incident from the left get further than negative spin particles, causing a positive chiral current which acts to enhance the Z condensate. This is the origin of the instability.

The fact that left and right handed fermions carry different charges now comes into play - particles of given S^z carry the same current when travelling in either direction! One can anticipate the destabilising effect - for positive Z there is a region where only $S^z > 0$ particles penetrate from the left, creating a positive chiral current. This acts through the equation of motion (2) to further destabilise the Z. Including currents from antiparticles, which are given by the substitutions $J \to -J$; $g_{A,V} \to -g_{A,V}$, multiplies the axial current by two, while cancelling the vector current. The parity violation in the Z-fermion coupling is crucial - the charged vector bosons $W^\pm$ do not destabilise the Z because the $Z/W^\pm$ equations of motion are parity invariant so that left and right moving $W^\pm$ modes carry opposite currents.

A more detailed analysis reveals a competing stabilising effect which exactly cancels the leading destabilising effect at zero wall velocity. The current carried per mode is given by the classical formula $J_z = -(\partial\omega/\partial Z)$ [17]. Integrating over momenta this is (for $S^z > 0$)

$$J_F = -\int \frac{d^3p}{(2\pi)^3} f(p) \frac{\partial \omega_+}{\partial Z} \tag{5}$$

$$= g_A \int \frac{d^3p_{-\infty}}{(2\pi)^3} \left(\frac{dp_z}{dp_{z,-\infty}}\right) f(p_{-\infty}) \frac{p_{z,-\infty}}{\sqrt{p_\perp^2 + p_{z,-\infty}^2}} \tag{6}$$

(the phase space density f is constant along particle trajectories). The only Z dependence is now in the Jacobian, which is just the ratio of the group velocity at infinity to that locally,

$$v_z = \frac{\partial\omega}{\partial p_z} = \frac{\sqrt{(p_{z,-\infty} + g_A Z)^2 - m^2}}{p_{z,-\infty} + g_A Z} \left(\frac{p_{z,-\infty}}{w}\right) \tag{7}$$

The Jacobian represents the enhancement of the local particle density due to a 'slowing down' effect. For positive Z, $S^z > 0$ particles see a 'well' and speed up, decreasing the chiral current, and $S^z < 0$ particles see a 'barrier', slowing down and adding a negative chiral current. Thus particles passing 'over' the barrier (in either direction) act to stabilise the Z condensate, whereas particles 'bouncing' off the barrier tend to destabilise it.

Now we compute the net chiral current in the WKB approximation. We assume the barrier is monotonic ($|(dZ/dm)| < 1$), and sum over particles and antiparticles of both spins, incident from both sides of the wall with thermal distributions at $z = \pm\infty$. As already mentioned, the leading effect, a square root divergence, cancels at zero v_w - there is a factor of two in the 'bouncing' contribution, cancelled by a minus one from particles going over the barrier in each direction. But at finite v_w more particles are encountered from the left, enhancing the number of 'bouncing' particles. The dominant term occurs as $m(z)$ approaches m_∞, and to lowest order in v_w is given by

$$J_F = \frac{2g_A N_C}{(2\pi)^2} \frac{m_\infty^2 v_w}{e^{\beta m_\infty} + 1} \sqrt{(m_\infty + g_A Z)^2 - m(z)^2} - (Z \to -Z) \tag{8}$$

where we sum over colors N_C. Note that

$$\frac{\partial J_F}{\partial Z}|_{Z=0} \propto g_A^2 m_\infty^3 (m_\infty^2 - m(z)^2)^{-\frac{1}{2}} \propto g_A^2 m_\infty^2 e^{z/2L} \tag{9}$$

diverging as $z \to \infty$, so that (within the WKB, free particle approximation we have made) the fermionic contribution to the effective Z squared mass diverges, due to the 'bouncing' effect, and an instability always develops sufficiently far behind the wall.

Several comments are in order. The destabilising term is proportional to m_∞^2, so the top quark dominates. We ignored the decay process $t \to W + b$, which occurs once the t gets a large enough mass on the wall. The rate for this process is $\frac{1}{16}\alpha_2 m_t^3/m_W^2(1 - m_W^2/m_t^2)^2(1 + 2m_W^2/m_t^2) \sim m_t/110$, and the timescale is long compared to the time the top quarks spend on the wall, in the regime of interest. We have treated the fermions bouncing off the wall as free particles, ignoring their interactions with the plasma. More detailed calculations are needed to reveal whether these interactions strengthen or suppress the destabilising term. One estimate is to assume the relevant top quarks are emitted with a thermal distribution a distance D away from the point z. Then in (9) the exponential is replaced by $e^{D/2L_w}$, with D a rather short diffusion length, smaller than L_w (we need the exponent to be ~ 3 for instability). But this is too pessimistic - it ignores the 'pile-up' of tops in front of the wall, particularly p_z below the barrier. In any case, a full semiclassical calculation including strong scattering in the Boltzmann equation appears quite feasible.

We emphasise that unlike the calculations of [6], [7] and [8] which are one dimensional, our calculations are three dimensional, and phase space is dominated by *large* perpendicular momenta $p_\perp \sim 2T$. At large p the quark damping rate [13] is smaller, and in the broken phase

the top's large mass makes it less sensitive to collisions at low momentum transfer. Note also that a difference from Farrar and Shaposhnikov is that the spontaneous CP violation here is essentially a *classical* effect, and does not rely on the quantum mechanical coherence. We expect it to be less susceptible to destruction by strong scattering.

The WKB calculation is indicative of an interesting effect, but since the instability occurs near (and because of!) a classical turning point, it is important to check it in a full quantum mechanical calculation. We used an exact solution to the Dirac equation in the background $\phi(z)^2 \propto (1+e^{-az})^{-1}$ in hypergeometric functions, in order to compute the full linear response kernel $K(x,y) \sim (\delta J_F(x)/\delta Z(y))$ [14], which enters the Z equation as

$$\ddot{Z}(z) = g_A^2\big(-4\phi^2 Z(z) + \frac{T^2 N_C}{4\pi^2}\int dy K(z,y)Z(y)\big) \quad (10)$$

The full quantum problem is nonlocal, and the condition for instability now reads

$$\frac{\int\int dxdy Z(x)K(x,y)Z(y)}{\int dx Z^2(x)(\phi^2(x)/\phi_\infty^2)} > \frac{16\pi^2\phi_\infty^2}{N_c T^2} \sim 100 \qquad (11)$$

for some trial function $Z(x)$, where we used the bound [2] $\phi_\infty \sim T/g_2 \sim 1.5T$ at the transition - if ϕ_∞ were smaller, baryons would not survive in the broken phase.

We have computed the integrals in (11) numerically, using Mathematica to evaluate the kernel. In order to focus on the 'bouncing' effect, we consider the case where the only particles incident are those below the barrier from the left, and we include both spins, particles and antiparticles. We integrate over $0 < p_\perp < \infty$ with a thermal distribution function and then numerically over $p_{z,-\infty}$, ignoring a phase space factor $x\mathrm{Log}(1+e^{-x})$, with $x = p_{z,-\infty}/T$ which is of order unity, and in any case could be enhanced by a chemical potential. We then compare the left hand side of (11) with the leading term in the WKB approximation for exactly the same quantity. (In WKB, the neglect of particles passing over the barrier has the effect of doubling the destabilising term).

In units where $T=1$, we used $a=.1$ (wall width $L_w = 10$), and $m_\infty = 1$ for the top mass in the broken phase. With a Gaussian trial function $\exp(-(x-x_0)^2/w^2)$, and $x_0 = 6L_W$, the left hand side of (11) is greatest for $w \approx 2L_W$, and has a value 179, compared to the WKB value of 161. The quantum problem thus shows a destabilising term even greater than the leading WKB approximation. A comprehensive set of results for different wall thicknesses will be presented in [14].

Several outstanding questions remain for future work. Exactly how does the CP violation from the KM matrix enter, and distinguish the 'in' and 'out' phases of the Z condensate? In a one dimensional treatment (which likely overestimates the effect), one can see from the calculations of [8] that a Z independent term $\sim d_{CP}$ enters in the equation of motion (2), and we expect to find similar CP violating terms at other even powers of Z. We are investigating the analogous three dimensional calculation. Finally, before the baryon asymmetry may be accurately calculated, we need to find the stable solutions for the Z condensate.

ACKNOWLEDGMENTS

We thank M. Bucher, D. Gross, I. Kogan, M. Joyce and T. Prokopec for discussions. This work was partially supported by NSF contract PHY90-21984, and the David and Lucile Packard Foundation.

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[15] We have confirmed these expectations in a calculation which will be described in [14].

[16] Consider a spherical bubble as in (Figure1): P leaves the Z configuration invariant, but C reverses it.

[17] In a box where the Z field is turned on adiabatically, $Z = Z(t)$, the canonical momentum of a mode doesn't change, but the work done is $-\int dt J_z \mathcal{E}_z$, with $\mathcal{E}$ the electric field. Using $\mathcal{E} = -\partial_t Z$ gives the wanted result.